Algebra Review Guide

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Linear Equations, Formulas and Inequalities

1. Eliminate denominators (multiply by Least Common Denominator)
2. Remove parentheses (distribute)
3. Get variable terms on one side (add/subtract principles)
4. Combine like terms (for formulas, factor the variable if it appears in more than 1 term)
5. Get the variable alone (multiply/divide principles)
   Note: For inequalities, division or multiplication by a
   negative number will switch the inequality symbol around
6. Check the solution

Linear Equations, Formulas and Inequalities

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Terminology

Vocabulary	Definition
Variable	A letter that can be replaced by any number
Constant	A capital letter that represents a fixed number
Expression	Consists of variables, numbers and operation symbols (ex: $2x - 5 + 3y^2 - x/3$)
Equation	When "=" is between two algebraic expressions (equation can be true $x + 2 = x + 2$, false $x + 2 = x + 3$ or neither $3x - 9 = 4x - 14$)
Solution	A value replacing the variable to make the equation true ($x = 5$ for $3x - 9 = 4x - 14$)
Solved	Having found the values that makes the equation true
Translate	To convert words to an algebraic expression or equation

Substitute	Replacing a variable with a given number
Evaluate	To find a solution
Factor	Multiple or Product
Factoring	Reversing distributive law and turning it into the factors (multiples)
Equivalent	Expressions that when evaluated, produce the same value
Terms	A number, var (variable) or a product/quotient of numbers and/or variables separated by + or – signs (Expression $5 - 3xy + 2x/5$ contains three terms)
Fraction notation	A way of showing division of two numbers (Numerator: top, Denominator: bottom)
Undefined	$a/0$: Division by zero is undefined — There is no solution
Zero fraction	$0/a$: Zero in the numerator makes fraction equal to zero
Fraction notation for 1	Any nonzero number divided by itself is 1: $a/a = 1$
Reciprocal	Multiplicative inverse: if $a \neq 0$, $a \cdot 1/a = a/b \cdot b/a = 1$ Zero has no reciprocals. The reciprocal of a is $1/a$.
Opposite	Additive inverse: opposite of "a" is "$-a$" and $a + (-a) = 0$
Prime number	A natural number that is divisible by only two different factors: itself and 1
Inequality	A statement about the relative size of two objects. Indicated by $<$, $>$, $\ll$, $\gg$
Law	Definition
Commutative	For any real numbers a and b, a + b = b + a and a • b = b • a
Associative	For any real numbers a, b and c, a • (b+c) = a • b + a • c; (a + b) + c = a + (b + c)
Distributive	For any real a, b and c, a + (b + c) = (a + b) + c and a • (b • c) = (a • b) • c

Operation with Fractions

Simplify

1. Prime factorize each numerator and denominator
2. Remove factors that are the same from numerator and denominator and replace with "1". These form fractions that are equal to "1".
3. Multiply the remaining factors in the numerator
4. Multiply the remaining factors in the denominator

Simplifying Fractions

Example

$60/126 = (2^2 \cdot 3 \cdot 5)/(2 \cdot 3^2 \cdot 7) = 2 \cdot 5/ 3 \cdot 7 = 10/ 21$

Multiply

1. Prime factorize each numerator and denominator
2. Remove factors that are the same from any numerator and any denominator and replace with "1". These form fractions that are equal to "1".
3. Multiply the remaining factors in all numerators
4. Multiply the remaining factors in all denominators

Multiplying Fractions

Example

$7/12 \cdot 30/21 = (7/2^2 \cdot 3) \cdot (2 \cdot 3 \cdot 5)/ (3 \cdot 7) = 1/2 \cdot 5/3 = 5/6$

Divide

1. Change division to multiplication by the inverse of the second fraction
2. Multiply as above

Dividing Fractions

Example

4/18 ÷ 10/21 = 4/18 • 21/10 = (2^2/2 • 3^2) • (3 • 7/2 • 5) = 1/3 • 7/5 = 7/15

Add and Subtract

1. If denominators are the same, go to step 4
2. Otherwise, prime factorize each denominator
3. Find Least Common Denominator (LCD) by multiplying each fraction by a fraction equal to "1", made out of the missing factors from each denominator
4. Once denominators are the same, add/subtract numerators and write over the LCD
5. Simplify as above

Adding/Subtracting Fractions

Example 1: Same denominator

2/14 + 5/14 = 7/14 = 7/(2 • 7) = 1/2

Example 2: One denominator a multiple of the other

3/2 − 5/6 = 3/2 − 5/2 • 3 = 3/2 • 3/3 − 5/2 • 3 = 9/2 • 3 − 5/2 • 3 = 9 - 5/ 2 • 3 = 4/2 • 3 = 2^2/2 • 3 = 2/3

Example 3: Different denominators

9/42 − 3/15 = 9/2 • 3 • 7 − 3/3 • 5 = 9/2 • 3 • 7 • 5/5 − 3/3 • 5 • 2 • 7/2 • 7 = 45 − 42/2 • 3 • 5 • 7 = 3 2 • 3 • 5 • 7 = 1/(2 • 5 • 7) = 1/70

Rules of Exponents

Zero and One

$a^0 = 1$

$a^1 = a$

Multiply and Divide

$a^b a^n = a^{b+n}$

$a^b/a^n = a^{b-n}$

Distribute

$(a^b c^n)^p = a^{bp} c^{np}$
$(a^b/c^n)^p = a^{bp} c^{np}$

Negative

$a^{-n} = 1/a^n$ $1/a^{-n} =$
a^n $a^{-n}/b^{-c} = b^c/a^n$

Rules of Radicals

Add and Subtract Radicals

Simplify each radical by removing perfect square roots out of the radical and to get like radicals then combine the number of like radicals.

Multiply and Divide Radicals

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

$\sqrt[n]{a} / \sqrt[n]{b} = \sqrt[n]{a/b}$

If no *n* is given, assume that *n* is 2 and a square root is required.

Plus/Minus Sign

Used to indicate that a value can be of either sign. Often used to construct confidence

intervals. $p \pm x + y = p + x + y$ *or* $p - x + y$

Data Transformation

TRIGONOMETRY AND NATURAL LOGARITHMS

Table 1

Transformation	Reciprocal of Transformation
Sine (sin)	Cosecant (csc or cosec) or Arcsine (arcsin)
Cosine (cos)	Secant (sec) or Arccosine (arccos)
Tangent (tan)	Cotangent (cot) or Arctangent (arctan)
Arcsine (arcsin)	Sine (sin)
Natural logarithm	Exponential (exp or e^n)
(In or log or $\log_e$)	

Summation

Given many terms of an infinite series or sequence, it is often desirable to find the sum of the terms. Such sums are easily expressed using the summation notation Σ (sigma) or S, which signifies that a series of terms should be added together. This is described below.

For an infinite series of numbers x_1, x_2, x_3, x_4, ….., x_n, the symbol

$$\sum_{i=1}^{n} x_i$$.

represents the sum of the first n terms in the series, that is, sum all x_i starting at $i = 1$ through $i = n$. Thus for a series with five term, that is, $n = 5$:

$$\sum_{i=1}^{5} x_i = x_1 + x_2 + x_3 + x_4 + x_5$$

Example 1: Given a series where $x_1 = 3$, $x_2 = 5$, $x_3 = 6$ and $x_4 = 2$, $x_5 = 7$,

$$\sum_{i=1}^{5} x_i = x_1 + x_2 + x_3 + x_4 + x_5 \quad = 3 + 5 + 6 + 2 + 7 = 23$$

Example 2: Let's say we have $a_i = i^2(i - 3)$. For n = 4, we calculate the summation by substituting, in succession, the integers 1, 2, 3, and 4 for i and adding the resulting terms as below.

$$\sum_{i=1}^{n} \quad i^2(i - 3) = 1^2(1 - 3) + 2^2(2 - 3) + 3^2(3-3) + 4^2(4 - 3) = (- 2) + (- 4) + 0 + 16 = 10$$

Multiplication

Multiplication Counting Principle

This principle makes is easier to solve a problem by organizing items being counted into arrays (rectangular or trees branches) and then multiplying to obtain totals. In a two choice scenario, if one choice can be made in x ways and a second choice can be made in y ways, then there are xy ways of making the first choice followed by the second choice.

Example: In a student's research project, there a plants with leaves that are either green, yellow-green, or yellow. The plants are either tall or dwarf, and contain enzyme A or enzyme B. Determine the possible combinations of leave color, plant height, and enzyme present.

Solution: Starting with leaf color, you can have:

- green/tall/enzyme A
- green/tall/enzyme B
- green/dwarf/enzyme A
- green/dwarf/enzyme B
- yellow-green/tall/enzyme A
- yellow-green/tall/enzyme B
- yellow-green/dwarf/enzyme A
- yellow-green/dwarf/enzyme B
- yellow/tall/enzyme A
- yellow/tall/enzyme B
- yellow/dwarf/enzyme A
- yellow/dwarf/enzyme B

That is 3 ways to choose leaf color by 2 ways to choose plant height by 2 ways to choose enzyme – 3 • 2 • 2 = 12

This can be solved using a tree diagram, and following the branches. So there are 12 possible choices.